

## **ALGORITHM FOR HIGH-PRECISION GEOREFERENCE AND RECTIFICATION OF HIGH-RESOLUTION SPACE IMAGES**

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### **Abstract**

*The current state-of-the-art of high-resolution space images enables the performance of large scale mapping, map updating, monitoring of the Earth cover and the environment, and other practical, scientific, and defence-related tasks requiring to determine with great accuracy the mutual position of individual discrete points [2, 3, 4, 5, 6, 7, 8, 9]. Based on a number of studies conducted at home and abroad [4, 7, 9], it was proven that the space images obtained upon their processing by the supplier and accordingly, upon their high-precision georeference and rectification by the user, differ substantially in their precision. This happens when the user determines the coordinates of the ground-based CPs using GPS measurements and dedicated software, accounting for the recommendations made during processing. Based on the designed mathematical model [1, 3, 5], an algorithm is suggested that may be used to prepare software for processing and assessment of collected observational material after the Least Square Method (LSM).*

**Key words:** *GPS, georeferenc, rectification, LSM*

### **1. Introduction**

In a series of studies [1, 2, 3, 5, 6], a mathematical model for georeference and rectification of high-resolution space images was developed. Based on this model, we shall present here an algorithm for preparation of software intended for processing of space images (scanner and photo images) and obtaining the coordinates of identified terrain control points (CPs) on the terrain and on the space image, and rectification of the geometric deformations.

Therefore, to ensure that high-resolution space images will accomplish their nowadays task in large-scale topographic mapping, map revision, monitoring of the environment, study of the outer space, ecology, safety, precise monitoring of earth cover changes etc., these images must be subject to preliminary processing [2,3,4,5,6,7, 9] comprising:

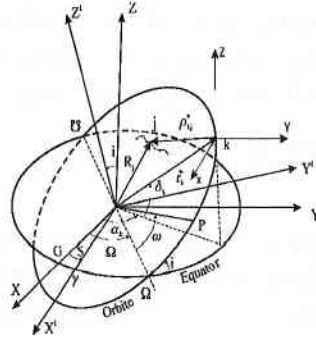
- high-precision coordinate georeference of the images of CPs, measured by GPS;
- rectification of the images by approximation functions, or by accounting for the changes in the scale factors along the scene's directions and the relief's configuration;
- using the Earth (reference) ellipsoid as a projection plane;
- accounting for the ellipsoid heights of the CPs;
- precise processing and result assessment after the Least Square Method (LSM).

The observance of these conditions will provide to obtain ultimate results with accuracy corresponding to the potentials of these modern space images.

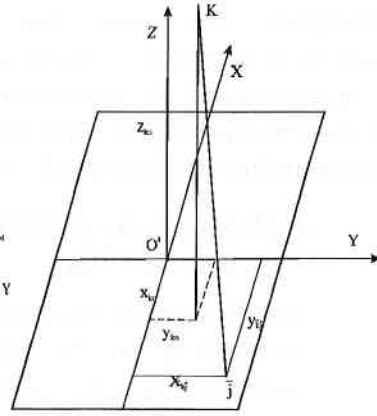
## **2. Orientation and stabilization of the space aircraft.**

The information which is obtained and used with space images is diverse with respect to both the elements to be defined and their location in space and time. For this reason, the coordinate systems in which the SA elements are determined refer to various space rectangular coordinate systems, namely:

1. *Greenwich Equatorial Geo-Centric Coordinate System -  $O, X, Y, Z$ .*
2. *Inertial Equatorial Geo-Centric Coordinate System -  $O, X', Y', Z'$ .*
3. *Geodetic Rectangular Geo-Centric Coordinate System -  $O, X_j, Y_j, Z_j$ .*
4. *Satellite-Centric Inertial Coordinate System -  $j - (x, y, z)_{kj}$  (Fig.1,2).*



**Fig. 1**



**Fig. 2**

According to Fig.1 we may write the following relation between the satellite-centric radius-vector  $\vec{\rho}_{kj}$ , the geo-centric radius-vector  $\vec{r}_k$ , and the topo-centric radius-vector  $\vec{R}_{kj}$  in coordinate form with respect to the *inertial geo-centric system (1)*, namely:

$$(1) \quad \vec{\rho}_{kj} = (\vec{R}_{kj} - \vec{r}_k) = \begin{bmatrix} X'_j - X'_k \\ Y'_j - Y'_k \\ Z'_j - Z'_k \end{bmatrix} = \rho_{kj} \begin{bmatrix} \cos \alpha_k \sin \delta_k \\ \sin \alpha_k \cos \delta_k \\ \sin \delta_k \end{bmatrix} = \rho_{kj} \begin{bmatrix} \xi_{kj} \\ \eta_{kj} \\ \zeta_{kj} \end{bmatrix},$$

where:

$$(2) \quad \xi_{kj}^2 + \eta_{kj}^2 + \zeta_{kj}^2 = 1;$$

$$\rho_{kj} = \sqrt{(X'_j - X'_k)^2 + (Y'_j - Y'_k)^2 + (Z'_j - Z'_k)^2}$$

$\alpha_{kj}$  and  $\delta_{kj}$  are the SA's rectascence and declination

$\vec{R}_j = (X', Y', Z')_j^T$  - the coordinates of the CP - j *in the inertial geo-centric system*

$\vec{r}_k = (X', Y', Z')_k^T$  - the coordinates of the SA - k *in the inertial geo-centric system.*

Let us assume that the vector  $\vec{D}_{kj}$  by the image  $\bar{j}$  on the space image (Fig.2), of terrain point j in the *inertial satellite-centric coordinate system* is expressed as follows:

$$(3) \quad \vec{D}_{kj} = \begin{bmatrix} x_{k\bar{j}} - x_{ko} \\ y_{k\bar{j}} - y_{ko} \\ z_{k\bar{j}} - z_{ko} \end{bmatrix} = D_{k\bar{j}} \begin{bmatrix} \xi_{kj} \\ \eta_{kj} \\ \zeta_{kj} \end{bmatrix},$$

$$D_{k\bar{j}} = \sqrt{(x_{k\bar{j}} - x_{ko})^2 + (y_{k\bar{j}} - y_{ko})^2 + (z_{k\bar{j}} - z_{ko})^2}$$

where:

$(x, y, z)_{kj}$  - coordinates of the CP's image -  $j$  on the space image;

$(x, y, z)_{ko}$  - coordinates of the photo's main point  $O$ , obtained by drawing a perpendicular from the object glass's back point.

Actually, the main point does not coincide with the origin  $O$  of the coordinate system on the space image (Fig.2).

From expressions (1) and (3), upon adequate solution, we obtain:

$$(4) \quad \vec{\rho}_{kj} = \rho_{kj} \begin{bmatrix} \xi_{kj} \\ \eta_{kj} \\ \zeta_{kj} \end{bmatrix} = \frac{1}{D_{k\bar{j}}} \rho_{kj} \vec{P}_o \begin{bmatrix} x_{k\bar{j}} - x_{ko} \\ y_{k\bar{j}} - y_{ko} \\ z_{k\bar{j}} - z_{ko} \end{bmatrix} = \begin{bmatrix} X_j - X_k \\ Y_j - Y_k \\ Z_j - Z_k \end{bmatrix}$$

Formulae (1)-(4) make it possible to determine the points from the space image in *satellite-centric inertial coordinate system*. But the coordinate georeference of the image must take place in the defined *Greenwich geo-centric coordinate system*, in which the satellite-centric radius-vector has the form:

$$(5) \quad \vec{\rho}_{kj} = \rho_{kj} \begin{bmatrix} \cos(\alpha_{kj} - S_k) \sin \delta_{kj} \\ \sin(\alpha_{kj} - S_k) \cos \delta_{kj} \\ \sin \delta_{kj} \end{bmatrix} = \rho_{kj} \begin{bmatrix} \xi_{kj} \\ \eta_{kj} \\ \zeta_{kj} \end{bmatrix} = \begin{bmatrix} X_j - X_k \\ Y_j - Y_k \\ Z_j - Z_k \end{bmatrix},$$

$S_k$  is the Greenwich star time, corresponding to the time  $t_k$  of receiving the space image. The coordinates of the SA  $-(X, Y, Z)_k$  and the CP  $-(X, Y, Z)_j$  are in the *Greenwich geo-centric coordinate system*.

From (4), the following relation may be written:

$$(6) \quad \begin{bmatrix} x_{k\bar{j}} - x_{ko} \\ y_{k\bar{j}} - y_{ko} \\ z_{k\bar{j}} - z_{ko} \end{bmatrix} = \frac{D_{kj}}{\rho_{kj}} \vec{P}_o^T \begin{bmatrix} X_j - X_k \\ Y_j - Y_k \\ Z_j - Z_k \end{bmatrix} = m \vec{P}_k \begin{bmatrix} X_j - X_k \\ Y_j - Y_k \\ Z_j - Z_k \end{bmatrix},$$

where:

$$(7) \quad m_{kj} = \frac{D_{kj}}{\rho_{kj}} \text{ - scale factor and}$$

$$(8) \quad \vec{P}_k = \vec{P}_o^T = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

The operator  $\vec{P}_k = \vec{P}_o^T$  is an orthogonal matrix, performing the transition from the *Greenwich* into the *satellite-centric geo-centric coordinate system* through the *Euler angles*  $(\Omega, w, i)$ , namely (Fig.1):

$$(9) \quad \begin{cases} a_1 = \cos w \cos \Omega - \sin w \sin \Omega \cos i, & b_1 = -\sin w \cos \Omega - \cos w \sin \Omega \cos i, \\ a_2 = \cos w \sin \Omega + \sin w \cos \Omega \cos i, & b_2 = -\sin w \sin \Omega + \cos w \cos \Omega \cos i, \\ a_3 = \sin w \sin i, & b_3 = \cos w \sin i, \\ c_1 = \sin \Omega \sin i, & c_2 = \cos \Omega \sin i, & c_3 = \cos i \end{cases}$$

From formula (6), according to formulae (7) and (8), we obtain:

$$(10) \quad \begin{cases} x_{k\bar{j}} = m_{kj}[a_1 \Delta X_{kj} + a_2 \Delta Y_{kj} + a_3 \Delta Z_{kj}] + x_{ko} = m_{kj} \bar{N}_{kj} + x_{ko} \\ y_{k\bar{j}} = m_{kj}[b_1 \Delta X_{kj} + b_2 \Delta Y_{kj} + b_3 \Delta Z_{kj}] + y_{ko} = m_{kj} \bar{P}_{kj} + y_{ko} \\ z_{k\bar{j}} = m_{kj}[c_1 \Delta X_{kj} + c_2 \Delta Y_{kj} + c_3 \Delta Z_{kj}] + z_{ko} = m_{kj} \bar{Q}_{kj} + z_{ko} \end{cases},$$

where we have:

$$\Delta X_{kj} = X_j - X_k, \quad \Delta Y_{kj} = Y_j - Y_k \quad \text{and} \quad \Delta Z_{kj} = Z_j - Z_k$$

$x_{kj}, y_{kj}, z_{kj}$  - coordinates of the images of the CP on the space image;

$x_{ko}, y_{ko}, z_{ko}$  - coordinates of the main point of the space image;

$X_j, Y_j, Z_j$  - geo-centric Greenwich coordinates of the CP from the Earth's surface;

$X_k, Y_k, Z_k$  - geo-centric Greenwich coordinates of the SA's projection centre

$a_i, b_i, c_i, \quad i = 1,2,3$  - elements of the orthogonal matrix  $\bar{P}_k$ , function of the Euler angles  $\Omega, w, i$

### 3. Preparation of an Algorithm to Determine the Equations of the Corrections and Unknown Variables for the Sought Quantities

To solve this problem, we shall start by assuming a number of settings and requirements, namely:

For each point  $\bar{j}$  of the space image, which appears to be the image of a CP- $j$  from the Earth's surface, we have twelve unknown variables, according to equations (10):

$$(11) \quad X_j, Y_j, Z_j, X_k, Y_k, Z_k, \Omega_k, w_k, i_k, x_{ko}, y_{ko}, z_{ko}$$

whereas we shall denote their approximate values in the following way:

$$(12) \quad X_j^o, Y_j^o, Z_j^o, X_k^o, Y_k^o, Z_k^o, \Omega_k^o, w_k^o, i_k^o, x_{ko}^o, y_{ko}^o, z_{ko}^o$$

Linearizing expressions (10) for each terrain control point  $j$  with coordinates  $\bar{j} = (x \ y \ z)_{kj}^T$ , projected onto the space image, the corrections equation is obtained:

$$(13) \quad \bar{V}_{U_{kj}} = \begin{pmatrix} \bar{A}_k & \bar{B}_k & \bar{C}_j & \bar{D}_{ko} \end{pmatrix} \begin{pmatrix} d\bar{S}_k \\ d\bar{r}_k \\ d\bar{R}_j \\ d\bar{n}_{ko} \end{pmatrix} + \bar{L}_{kj}; \quad P_{kj}$$

$P_{kj}$  - weighing factor

The quantities  $\bar{A}_k, \bar{B}_k, \bar{C}_j, \bar{D}_{ko}$  in the corrections equation (10) must be considered as partial differentials of the coordinates  $x_{kj}, y_{kj}, z_{kj}$ , namely :

$$(14) \quad \bar{A}_k = \frac{\partial(x, y, z)_{kj}}{\partial(\Omega, w, i)_{kj}}; (15) \quad \bar{B}_k = \frac{\partial(x, y, z)_{kj}}{\partial(X, Y, Z)_{k(j)}}; (16) \quad \bar{D}_{ko} = \frac{\partial(x, y, z)_{kj}}{\partial(x, y, z)_{ko}}$$

whereas  $\vec{B}_k = -\vec{C}_j$ , the index "k" is differentiation along the coordinates of the SA, and the index "j" - differentiation along the coordinates of a CP from the terrain.

$$(17) \quad \begin{cases} \vec{V}_{U_n} = (v_x \ v_y \ v_z)_{kj}^T; & d\vec{S}_k = (d\Omega \ dw \ di)_k^T; \\ d\vec{r}_j = (dX \ dY \ dZ)_k^T; & d\vec{R}_j = (dX \ dY \ dZ)_j^T; \\ d\vec{n}_{ko} = (dx \ dy \ dz)_{ko}^T \end{cases}$$

$d\vec{S}_k, d\vec{r}_k, d\vec{R}_j, d\vec{n}_{ko}$  are the corrections for the approximate values (12) of the known (10) and unknown variables (11).

For the vector of the free term  $\vec{L}_{kj}$  we have:

$$(18) \quad \vec{L}_{kj} = \vec{U}_{kj} - \vec{U}'_{kj} = \begin{vmatrix} x_{kj} - x'_{kj} \\ y_{kj} - y'_{kj} \\ z_{kj} - z'_{kj} \end{vmatrix},$$

where:

$\vec{U}_{kj} = (x \ y \ z)_{kj}^T$  - the determined values for the coordinates  $x_{kj}, y_{kj}, z_{kj}$  after (10),

$\vec{U}'_{kj} = (x' \ y' \ z')_{kj}^T$  - the measured coordinates from the space image.

#### 4. Deriving expressions to determine the values $\vec{A}_k, \vec{B}_k, \vec{C}_j$

To obtain the private differentials according to expressions (14), (15) and (16), the following coordinates must be successively differentiated:  $x_{kj}, y_{kj}, z_{kj}$  with respect to the Euler angles  $(\Omega, w, i)$ ; the space coordinates of the terrain CP-j with respect to  $(X \ Y \ Z)_j$ , the Greenwich coordinates of the SA with respect to  $(X \ Y \ Z)_k$  and also with respect to the coordinate origin of the image  $(x \ y \ z)_{ko}$ .

#### 4.1. Private differentials for the quantity $\vec{A}_k$

According to expression (14), it is not necessary to differentiate the image coordinates from (10) with respect to  $(\Omega, w, i)$ , accordingly - (9). But since only the quantities  $a_i, b_i, c_i, (i = 1,2,3)$  are function of the Euler angles, we must differentiate  $\bar{N}_{kj}, \bar{P}_{kj}, \bar{Q}_{kj}$  by the expressions:

$$(19) \left\{ \begin{array}{l} \frac{\partial x_{\bar{k}j}}{\partial(\Omega, w, i)_k} = \frac{\partial(m_{kj} \bar{N}_{kj})}{\partial(\Omega, w, i)_k}; \quad \frac{\partial y_{\bar{k}j}}{\partial(\Omega, w, i)_k} = \frac{\partial(m_{kj} \bar{P}_{kj})}{\partial(\Omega, w, i)_k}; \quad \frac{\partial z_{\bar{k}j}}{\partial(\Omega, w, i)_k} = \frac{\partial(m_{kj} \bar{Q}_{kj})}{\partial(\Omega, w, i)_k} \end{array} \right.$$

#### 4.2. Private differentials for the quantities $\vec{B}_k = -\vec{C}_j$

As already mentioned above, to obtain the differentials of the image coordinates  $(x, y, z)_{\bar{k}j}$  with respect to  $(X \ Y \ Z)_k^T$  and  $(X \ Y \ Z)_j^T$ , we must use expressions (10), from which it follows that both the scale  $m_{kj} = \frac{D_{kj}}{\rho_{kj}}$ , according to formula (7), and  $\bar{N}_{kj}, \bar{P}_{kj}, \bar{Q}_{kj}$  are function of the Greenwich coordinates. Accounting for this fact, we shall differentiate by expressions

$$(20): \left\{ \begin{array}{l} \frac{\partial x_{\bar{k}j}}{\partial(X, Y, Z)_{k(j)}} = \frac{\partial(m_{kj} \bar{N}_{kj})}{\partial(X, Y, Z)_{k(j)}}; \quad \frac{\partial y_{\bar{k}j}}{\partial(X, Y, Z)_{k(j)}} = \frac{\partial(m_{kj} \bar{P}_{kj})}{\partial(X, Y, Z)_{k(j)}}; \quad \frac{\partial z_{\bar{k}j}}{\partial(X, Y, Z)_{k(j)}} \\ = \frac{\partial(m_{kj} \bar{Q}_{kj})}{\partial(X, Y, Z)_{k(j)}} \end{array} \right.$$

### 5. Determination of the elements of the private differentials during coordinate georeference of space images

#### 5.1. Formulae for the private differentials of the coordinates $(x, y, z)_{\bar{k}j}$ with respect to the Euler angles $(\Omega, w, i)_k$

According to expressions (19), in order to obtain the elements of the differentials of linear equation system (10) with respect to  $a_i, b_i, c_i, (i = 1,2,3)$ , we must differentiate (9) with respect to the Euler angles  $(\Omega, w, i)_k$ , which results in:



$$(21) \quad \begin{array}{c} \frac{\partial x_{kj}}{\partial \Omega_k} \\ \frac{\partial x_{kj}}{\partial w_k} \\ \frac{\partial x_{kj}}{\partial i_k} \end{array} = m_{kj} \begin{array}{c} \frac{\partial \bar{N}_{kj}}{\partial \Omega_k} \\ \frac{\partial \bar{N}_{kj}}{\partial w_k} \\ \frac{\partial \bar{N}_{kj}}{\partial i_k} \end{array} = m_{kj} \begin{array}{c} -a_2 \quad a_1 \quad 0 \\ b_1 \quad b_2 \quad b_3 \\ c_1 \sin \omega_k \quad -c_2 \sin \omega_k \quad c_3 \sin w_k \end{array} \begin{array}{c} \Delta X_{kj} \\ \Delta Y_{kj} \\ \Delta Z_{kj} \end{array}$$

$$(22) \quad \begin{array}{c} \frac{\partial y_{kj}}{\partial \Omega_k} \\ \frac{\partial y_{kj}}{\partial w_k} \\ \frac{\partial y_{kj}}{\partial i_k} \end{array} = m_{kj} \begin{array}{c} \frac{\partial y_{kj}}{\partial \Omega_k} \\ \frac{\partial y_{kj}}{\partial w_k} \\ \frac{\partial y_{kj}}{\partial i_k} \end{array} = m_{kj} \begin{array}{c} -b_2 \quad b_1 \quad 0 \\ -a_1 \quad -a_2 \quad -a_3 \\ b_3 \sin \Omega_k \quad -b_3 \cos \Omega_k \quad c_4 \end{array} \begin{array}{c} \Delta X_{kj} \\ \Delta Y_{kj} \\ \Delta Z_{kj} \end{array}$$

$$(23) \quad \begin{array}{c} \frac{\partial z_{kj}}{\partial \Omega_k} \\ \frac{\partial z_{kj}}{\partial w_k} \\ \frac{\partial z_{kj}}{\partial i_k} \end{array} = m_{kj} \begin{array}{c} \frac{\partial \bar{Q}_{kj}}{\partial \Omega_k} \\ \frac{\partial \bar{Q}_{kj}}{\partial w_k} \\ \frac{\partial \bar{Q}_{kj}}{\partial i_k} \end{array} = m_{kj} \begin{array}{c} -c_2 \quad c_1 \quad 0 \\ 0 \quad 0 \quad 0 \\ c_3 \sin \Omega_k \quad -c_3 \cos \Omega_k \quad -c_4 = \sin i_k \end{array} \begin{array}{c} \Delta X_{kj} \\ \Delta Y_{kj} \\ \Delta Z_{kj} \end{array}$$

5.2. Formulae for the private differentials of the coordinates  $(x, y, z)_{kj}$  with respect to  $(X, Y, Z)_{k(j)}$  at the time of receiving the image  $t_k$

According to expressions (20), the elements of the private differentials may be obtained from expressions (10) with respect to the Greenwich coordinates of the SA -  $(X, Y, Z)_{k(j)}$  at the time of exposure of the space image  $t_k$ , namely:

$$(24) \quad \begin{cases} \frac{\partial x_{k(j)}}{\partial (X, Y, Z)_{K(j)}} = \pm m_{K(j)} \left[ \frac{\bar{N}_{KJ}}{\rho^2_{KJ}} (X_{KJ} + \Delta Y_{KJ} + \Delta Z_{KJ})^T \mp (a_1 + a_2 + a_3)^T \right] \\ \frac{\partial y_{k(j)}}{\partial (X, Y, Z)_{K(j)}} = \pm m_{K(j)} \left[ \frac{\bar{P}_{KJ}}{\rho^2_{KJ}} (\Delta X_{KJ} + \Delta Y_{KJ} + \Delta Z_{KJ})^T \mp (b_1 + b_2 + b_3)^T \right] \\ \frac{\partial z_{k(j)}}{\partial (X, Y, Z)_{K(j)}} = \pm m_{K(j)} \left[ \frac{\bar{Q}_{KJ}}{\rho^2_{KJ}} (\Delta X_{KJ} + \Delta Y_{KJ} + \Delta Z_{KJ})^T \mp (c_1 + c_2 + c_3)^T \right] \end{cases}$$

In expressions (24), for the indexes k and j we have accordingly k = 1, 2, ..., 9 and j = 1, 2, ..., 9.

For the matrix  $C_j$  the values are the same, but the signs change, i.e., "+" becomes "-" and "-" becomes "+". In the above expressions, we have assumed that  $\Delta x_{oj}, \Delta y_{oj}, \Delta z_{oj}$  are equal to:

$$(25) \quad \Delta x_{oj} = x_{kj} - x_{ko}; \Delta y_{oj} = y_{kj} - y_{ko}; \Delta z_{oj} = z_{kj} - z_{ko}$$

The values of these quantities are determined from (14), i.e., these are the calculated coordinates of the images of the CPs.

The scale factor  $m_{kj}$  is determined from formula (10) for all terrain CP and their respective images on the space image.

5.3. Expression to determine the private differentials  $(x, y, z)_{kj}$  with respect to  $(x, y, z)_{ko}$

According to expression (16) and linear equation system (10), and taking into account that according to equation (10), in determining  $m_{kj}$  the distance  $D_{kj}$  from the image is used, formula (3), which is function of the coordinates of the main point  $x_{ko}, y_{ko}, z_{ko}$  of the image O assumes the form:

$$(25) \quad D_{ko} = \begin{vmatrix} \frac{(x_{kj} - x_{ko})^2}{D^2_{ko}} & 0 & 0 \\ 0 & \frac{(y_{kj} - y_{ko})^2}{D^2_{ko}} & 0 \\ 0 & 0 & \frac{(z_{kj} - z_{ko})^2}{D^2_{ko}} \end{vmatrix}$$

The essential thing here is that the scale coefficient  $m_{kj}$  is calculated for each terrain CP, thereby providing us with the image deformation between each particular image point and the origin of the coordinate system.

The vector equation for the corrections (13) and the obtained values for the private differentials based on expressions (21)–(24) make it possible to present the corrections of the sought quantities and the corrections for the unknown variables in the following form:

(26)

$$V_{xkj} = A_1 d\Omega + A_2 d\omega + A_3 dt + K_1 dX_k + K_2 dY_k + K_3 dZ_k + J_1 dX_j + J_2 dY_j + J_3 dZ_j + dx_{k0} + l_{xkj}$$

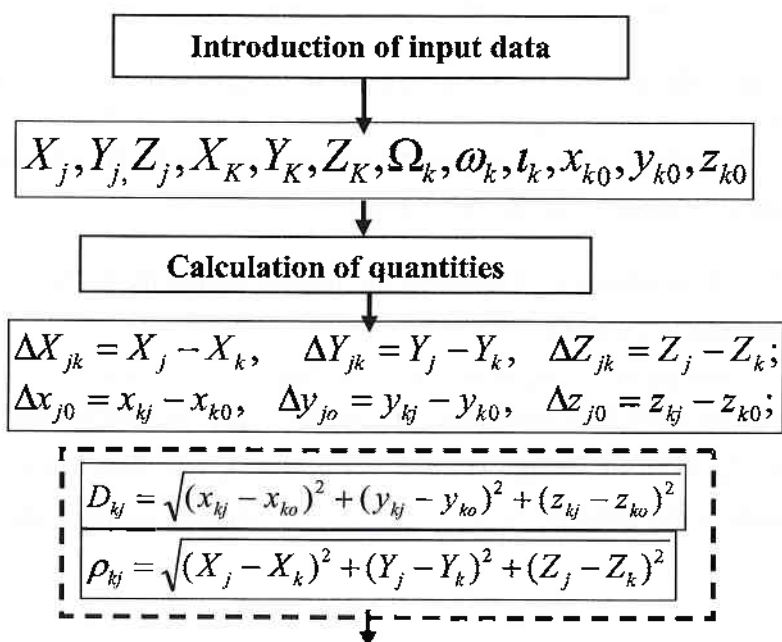
(27)

$$V_{y_{kj}} = B_1 d\Omega + B_2 d\omega + B_3 dt + K_4 dX_k + K_5 dY_k + K_6 dZ_k + J_4 dX_j + J_5 dY_j + J_6 dZ_j + dy_{k0} + l_{y_{kj}}$$

(28)

$$V_{z_{kj}} = C_1 d\Omega + C_2 d\omega + C_3 dt + K_7 dX_k + K_8 dY_k + K_9 dZ_k + J_7 dX_j + J_8 dY_j + J_9 dZ_j + dz_{k0} + l_{z_{kj}}$$

## 6. Calculation algorithm for the coordinates' georeference after the suggested mathematical model



$$\begin{cases} a_1 = \cos w \cos \Omega - \sin w \sin \Omega \cos i, & b_1 = -\sin w \cos \Omega - \cos w \sin \Omega \cos i, \\ a_2 = \cos w \sin \Omega + \sin w \cos \Omega \cos i, & b_2 = -\sin w \sin \Omega + \cos w \cos \Omega \cos i, \\ a_3 = \sin w \sin i, & b_3 = \cos w \sin i, \\ c_1 = \sin \Omega \sin i, & c_2 = \cos \Omega \sin i, & c_3 = \cos i, & -c_4 = \sin i_k \end{cases}$$

$$\begin{cases} \bar{N}_{kj} = a_1 \Delta X_{kj} + a_2 \Delta Y_{kj} + a_3 \Delta Z_{kj} \\ \bar{P}_{kj} = b_1 \Delta X_{kj} + b_2 \Delta Y_{kj} + b_3 \Delta Z_{kj} \\ \bar{Q}_{kj} = c_1 \Delta X_{kj} + c_2 \Delta Y_{kj} + c_3 \Delta Z_{kj} \end{cases}$$

**Calculation**  
**corrections equation (26) – (28)**  
**and factors equation (21) – (24)**

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## **АЛГОРИТЪМ ЗА ВИСОКОТОЧНО ПРИВЪРЗВАНЕ И РЕКТИФИКАЦИЯ НА КОСМИЧЕСКИ ИЗОБРАЖЕНИЯ С ВИСОКА РАЗДЕЛИТЕЛНА ВЪЗМОЖНОСТ**

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### **Резюме**

Сегашното състояние на космическите изображения с висока разделителна способност, дават реална възможност за едромащабно картиране, обновление на съществуващи карти, мониторинг на земното покритие и окръжаващата среда и други практични, научни и отбранителни цели, при които е необходимо да се постигат високи точности при определяне взаимното разположение между отделни дискретни точки. [2,3,4,5,6,7,8,9]

На базата на многото изследвания у нас и в чужбина [4,7,9] се доказва, че резултатите от обработката на космическите изображения от доставчика и ползвателя, се различават чувствително по точност при привързването и ректификацията на изображенията. Това се получава когато ползвателя определя координатите на земните ОТ с GPS измервания и съответно вземе под внимание направените препоръки при обработката.

На основата на създадения математически модел [1,3,5] се разработи алгоритъм по който се изготвя софтуер за обработка и оценка на получения наблюдателен материал по метода на най-малките квадрати (МНК).